Optimum Source Conductance for High Frequency Superconducting Quasiparticle Receivers

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***Abstract*— We have used the quantum theory of mixing for extensive numerical calculations to determine the mixer source conductance G_s required to optimize a superconductor-insulator-superconductor (SIS) quasiparticle heterodyne receiver. The optimum G_s matches an empirical formula which can be understood by a simple derivation. Previous work indicated that G_s should vary inversely with frequency, and this implies that the critical current density of SIS junctions used for mixing should increase as frequency *squared*, a stringent constraint on the design of submillimeter SIS mixers. On the contrary, we find that G_s is more weakly dependent upon frequency, and we discuss the implications for the design of submillimeter SIS mixers.**

I. Introduction

SUPERCONDUCTOR-insulator-superconductor (SIS) quasiparticle mixers [1] are now firmly established as the most sensitive receiving devices in the vicinity of 100 to 200 GHz. Their behavior is well described by Tucker's quantum theory of mixing [2]. There are now many publications which show excellent agreement between the theory's predictions of a mixer's conversion properties and experimental results, especially at 100 GHz, and the theory also appears to be successful in predicting the noise temperature of the most sensitive SIS mixers.

At higher frequencies there are fewer experimental results, but many experiments are underway and low noise SIS receivers have been demonstrated at frequencies as high as 500 GHz. Given the success of the Tucker theory at 100 GHz, it is desirable to know the predicted performance of SIS mixers at higher frequencies. In particular, it is desirable to predict the optimum value of the characteristic parameter $\omega R_N C$, where ω is the LO frequency and R_N is the normal state resistance and C the capacitance of the SIS junction, and also to predict the optimum value of the quantity $G_s R_N$, the mixer source conductance normalized to R_N. These parameters should be established early in the design of an SIS mixer so that the SIS junction area and the mixer block source admittance can be chosen.

The first systematic attempt to do this using realistic experimental constraints was made by Kerr and Pan [3], who developed a "design procedure" for SIS mixers, really a set of rules for scaling a successful and reasonably understood low-frequency SIS mixer design to higher frequency. Their analysis, which was carried further and ratified in [4], was based on the "$\omega R_N C = 4$ rule": the best SIS mixer performance near 100 GHz appears to be obtained when $\omega R_N C$ is near 4. As first advanced in [5] and more recently discussed in [6], all SIS mixer experiments exhibiting infinite available gain have $\omega R_N C \gtrsim 4$, while $\omega R_N C < 1$ has always resulted in considerable conversion loss. (To our knowledge this correlation still holds to date.) Presumably, good mixer conversion requires the reduction of harmonic conversion effects by the relatively large capacitance. Indeed, computer simulations show that harmonic conversion becomes significant for $\omega R_N C < 4$ [7]. On the other hand, unnecessarily large capacitance entails greater difficulty in tuning and narrower bandwidth.

The time constant $R_N C$ of an SIS junction varies in inverse proportion to its critical current density, J_c. To maintain a constant $\omega R_N C$, therefore, J_c must increase proportional to frequency and this alone requires an inconveniently large J_c for submillimeter SIS mixers. However, Kerr and Pan rightly noted that while the $\omega R_N C = 4$ rule may be valid for 100 GHz SIS mixers, there is no reason to expect that the optimum $\omega R_N C$ is independent of frequency. In particular, their computer simulations indicated that the normalized source conductance $G_s R_N$ should vary inversely with frequency for mixers in the quantum-limited regime. Their derivation then showed that $\omega R_N C$ should also vary inversely with frequency for a frequency-scaled mixer block, which immediately implies that J_c should increase as frequency *squared.* This result presents a stringent constraint on the design of submillimeter SIS mixers. For instance, high quality Nb/Al_2O_3/Nb junctions require $J_c \approx 1500$ A/cm^2 for $\omega R_N C = 4$ at 100 GHz, a good match to the standard J_c from various Josephson junction foundries. But then a frequency-scaled 500 GHz SIS mixer should have $J_c \approx 40\,000$ A/cm^2, much higher than can be conveniently fabricated. Thus, [3] implies that high frequency SIS mixers are much more difficult to realize than has previously been appreciated.

We disagree with this conclusion. We find that $G_s R_N$ is only weakly dependent upon frequency for high frequency SIS mixers. We present an empirical formula for the optimum $G_s R_N$ which is an excellent fit to our extensive computed data, and then explain that formula by a simple derivation.

II. Calculations

It is not feasible to optimize an SIS mixer by maximizing the calculated conversion gain. There is no unique optimum bias point: the quantum theory of mixing predicts infinite gain

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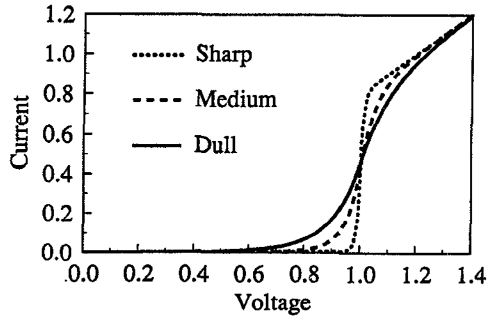

Fig. 1. Three synthetic normalized I-V characteristics used for calculations.

for high quality SIS junctions over a wide range of parameter values. Such high gain is unrealistic and undesirable. Kerr and Pan avoided this difficulty by positing a set of requirements, including unity gain and moderately well matched input (VSWR $\lesssim 2$), for optimum mixer design. We take a different approach.

We use the quantum mixer theory for extensive numerical calculations, to determine the minimum value of the SSB (single sideband) noise temperature T_R of an SIS receiver, subject to reasonable experimental constraints. Thus our calculation involves a trade-off between minimizing the mixer noise temperature and maximizing the mixer conversion gain. This is mediated by the noise temperature $T_{\rm IF}$ of the IF amplifier, which is assumed to include an input isolator. We take the signal and image embedding impedances seen by the SIS junction to be equal, we make the standard three-frequency low-IF approximation, we ignore all reactances including the quantum susceptance (see below), and we do not include any interference from the Josephson effect. This model is equivalent to assuming 1) that the geometrical capacitance of the SIS junction is large enough to both short out the LO harmonics and their sidebands and to eliminate any Josephson interference, and 2) that the capacitance is itself resonated by a relatively broadband external tuning circuit, so that the linear circuitry is purely resistive at all relevant frequencies.

Rather than specify a particular experimental SIS I-V curve for the simulations, we use two-parameter synthetic I-V curves to examine the influence of junction quality. The synthetic I-V curves look very much like real junction I-V curves, and cover the range of junctions (and of operating temperature) used for SIS mixing. The data presented here use the curves depicted in Fig. 1. (We normalize voltages to the energy gap voltage V_g and conductances to the normal state resistance R_N.) We assume the entire current is due to single particle tunneling.

A novel aspect of our calculation is that we require both the signal reflection gain and also the signal-to-image conversion gain to be small ($\leq 1/4$, which corresponds to VSWR ≤ 3), to obtain a reasonable input match. We find our quantitative results are extremely insensitive to the level of returned signal or image power allowed. This interesting topic will be explored at length in a future publication [8]

The quantum susceptance of an SIS mixer certainly influences the shape of the pumped I-V curve, and widens the range of possibility of infinite gain [9]. However, the quantum susceptance appears to have little effect on the performance of our optimized SIS receiver. This is because whenever the mixer is biased to enable the quantum susceptance to increase the IF conversion gain, it causes the returned signal and image powers to increase even more rapidly; the net effect is to degrade the receiver performance. This issue will be more fully discussed in [8].

Finally, the three contributions of thermal noise to T_R are treated as follows. 1) The thermal noise from the image termination does not influence the optimization of our receiver; it simply adds to T_R. Therefore we assume zero physical temperature for the image termination without loss of generality. 2) Our computer simulations show that the thermal component of the shot noise (in the current correlation matrix) is negligible for optimized SIS mixers. 3) Thermal noise arising in the IF isolator is reflected from the mixer back into the IF amplifier. Simulations with the IF isolator at 4.2 K show that this noise has an almost imperceptible effect on receiver optimization for all realistic parameter values. Therefore the IF-reflected thermal noise is considered a component of $T_{\rm IF}$.

Our model should be a fairly good representation of the best experimental SIS receivers. We maintain that it is the simplest possible model which exhibits the essence of optimized SIS mixer behavior. Other phenomena not included here can best be studied and understood as extensions of the model once the properties of this simple model are known.

The equations employed to calculate T_R are taken from [1]. At each frequency the optimum value of G_s, and of the amplitudes of the dc bias and the LO voltage across the junction, are determined given discrete values for the remaining parameters. The discrete parameters are $T_{\rm IF}$, the IF load conductance G_L, and two parameters characterizing the SIS junction I-V curves. We have performed these calculations for a wide range of the discrete parameters, covering the complete range of experimental SIS receivers. The subset of data chosen to present here are fully illustrative of our complete results.

III. Results

Fig. 2 shows the minimum theoretical SSB noise temperature of our model SIS receiver with $G_L = 0.3/R_N, T_{\rm IF} = 3\,{\rm K}$, and $V_g = 3\,{\rm mV}$, for the three I-V curves of Fig. 1 (frequency is normalized to the energy gap frequency $\omega_g \equiv eV_g/\hbar$). Fig. 3 shows the optimum value of normalized source conductance G_sR_N required to achieve the minimum T_R. At lower frequencies (below the vertical rise in each curve) the mixer is biased on higher number photon steps and G_s is relatively constant as expected for classical behavior. On the first photon step, however, the behavior of G_s is quite different. At the lowest frequencies on the first step G_s strongly depends on the junction quality; for high quality junctions the optimum G_s is rather large. As the frequency increases, the optimum G_s gradually changes to approach a value ≈ 0.7, for all three I-V curves at frequencies near $2\omega_g$.

In Fig. 4 we replot the same data as in Fig. 3 in terms of the normalized quantities $G_s\omega$ versus ω. Fig. 4 clearly shows that the optimum G_s for the sharp curve is matched by the empirical formula

$$G_s = \frac{1}{2} + \frac{1}{4\omega}, \tag{1}$$

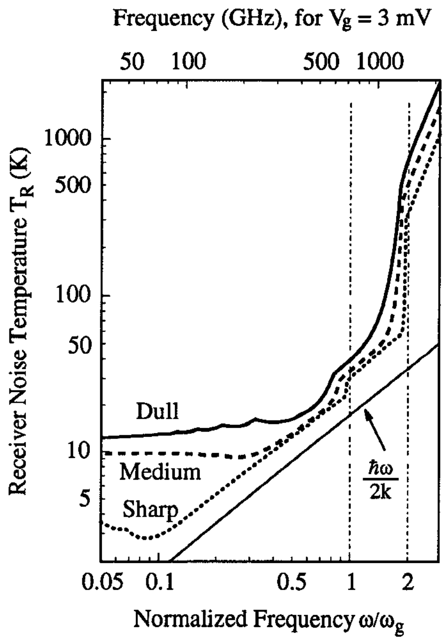

Fig. 2. The SSB noise temperature of our model SIS receiver optimized at each frequency, calculated for the three I-V curves of Fig. 1, $G_L = 0.3/R_N, T_{\mathrm{IF}} = 3$ K, and $V_g = 3$ mV.

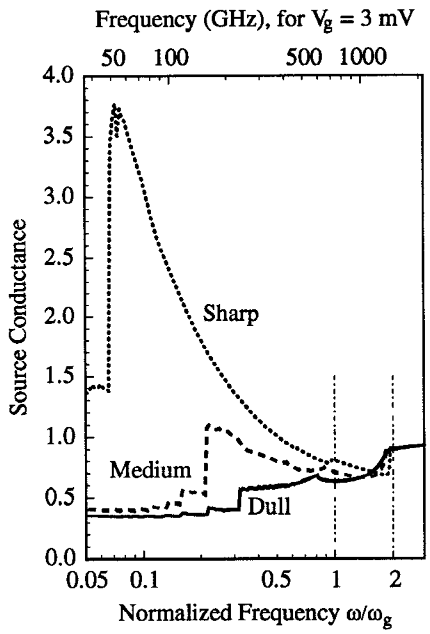

Fig 3. The normalized source conductance $G_s R_N$ required to optimize the receiver of Fig. 2, calculated for the three I-V curves of Fig. 1.

for bias points on the first photon step. This behavior is quite widespread: The same empirical formula also works well when we consider different values of G_L or T_{IF}, I-V curves with considerable leakage current, etc. For instance, Fig. 5 shows the optimum G_s computed for SIS receivers with various values of G_L, for the sharp I-V curve. Equation (1) demonstrates that while G_s may approximate a $1/\omega$ dependence for low frequency, at higher frequencies the optimum G_s is roughly independent of frequency. Note that $G_s \sim 1/\omega$ would give horizontal lines in Figs. 4 and 5.

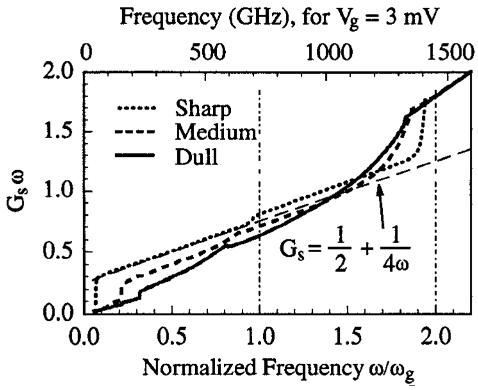

Fig. 4. The data of Fig. 3 are multiplied by ω and replotted (in normalized units), and are compared to an empirical formula.

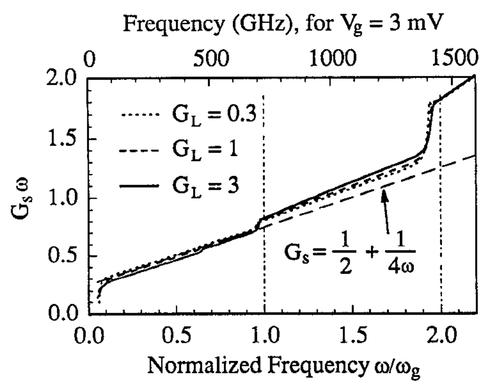

Fig. 5. The optimum source conductance (in normalized units) of an SIS receiver whose IF load conductance $G_L = 0.3/R_N, 1.0/R_N$, and $3.0/R_N$, respectively, using the "sharp" I-V curve of Fig. 1, $T_{\mathrm{IF}} = 3$ K, and $V_g = 3$ mV.

In order to better understand the behavior of the optimum G_s we present Fig. 6, in which we compare G_s with all of the important "input" conductances we can calculate. It is seen that even though G_s is determined by a trade-off between the gain and the shot noise, the optimum G_s is quite close to that which minimizes the shot noise, G_{shot}, but far from that which maximizes the gain, G_s'. This surprising result can be explained by examination of the equations of the SIS mixer. On one hand, the dependence of the conversion gain upon G_s is given by a simple impedance matching formula which has its maximum at $G_s = |G_s'|$; a fairly large mismatch therefore results in only a small decrease in gain. On the other hand, the mixer noise is minimized by the exact cancellation of the correlated components of the shot noise at the IF and the signal and image frequencies, which occurs at $G_s = G_{\mathrm{shot}}$. If G_s strays from this value the shot noise grows rapidly. The optimum G_s is also rather far from the signal input conductance, G_s°, but never more than a factor of three or else the signal reflection gain is greater than 1/4, which we do not allow.

Note in Fig. 6 that G_{shot}, and thus the optimum G_s, follows closely the input conductance at the LO frequency, G_{LO}°. Surprisingly, this is exactly as predicted by the simple photodiode theory of SIS mixing [10], which reproduces the equations of Tucker's quantum theory of mixing in the limit of

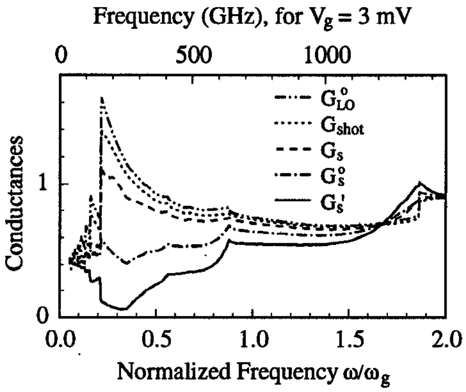

Fig. 6. The optimum source conductance G_s of an SIS receiver using the "medium" I-V curve of Fig. 1, $G_L = 0.3/R_N, T_{\rm IF} = 3$ K, and $V_g = 3$ mV, compared to various "input" conductances (all normalized to R_N) : $G^{\circ}_{\rm LO}$ and G°_s are the input conductances of the mixer at the LO and the signal frequencies, respectively, $G_{\rm shot}$ is the value of G_s which would minimize the shot noise of the mixer, and G'_s is the value of G_s which would maximize the gain of the mixer.

small LO voltage amplitude (small α). In our simulations as in real SIS mixers the optimum value of α is relatively large, on the order of unity, and so the small α limit should at best be a rough approximation. In any case this enables us to understand the empirical formula (1). In the limit of small $\alpha, G^{\circ}_{\rm LO}$ is the slope of the chord connecting the photon point $I_{\rm dc}(V_0 - \hbar\omega/e)$ to the photon point $I_{\rm dc}(V_0 + \hbar\omega/e)$ on the unpumped dc I-V curve. Therefore, using the preferred value (found in our simulations) for the optimum dc bias voltage $V_0 = 0.9$ for the sharp I-V curve, this gives $G^{\circ}_{\rm LO} = 1/2 + 0.35/\omega$ in the small α limit. (Note in Fig. 1 that the "normal" current extrapolates to $V = 0.2$ at $I = 0$.) Gs follows but is slightly less than $G^{\circ}_{\rm LO}$ (Fig. 6) and so G_s is very well approximated by (1).

IV. Discussion

The results presented here are for particular parameter values, but they are quite general and representative of our more extensive calculations. We find that the optimum value of G_s for an SIS receiver using high quality junctions is given by (1). Thus G_s is only weakly dependent upon frequency for high frequency SIS mixers. For a receiver using lesser quality SIS junctions the optimum value of G_s is even less frequency dependent. These results disagree with the conclusions stated in [3]; therefore J_c need not increase as frequency squared for optimum high frequency SIS mixers.

Note that the numerical simulations presented in [3], for normalized frequency less than 0.5, are in fact in rough accord with (1). The remaining disagreement likely arises because [3] fixed the gain to unity, while for our optimized SIS receiver the gain falls off roughly as $1/\omega$ for bias points on the first photon step (Fig. 7). Fig. 7 shows conversion gain as high as 8 dB in the vicinity of 100 GHz, with low noise and quite low returned signal and image power.

Equation (1) could be used to generate a formula giving a "best" value for $\omega R_N C$ and hence for J_c, but we believe this is inappropriate: the "$\omega R_N C = 4$ rule" may be valid at 100 GHz but should be discarded for submillimeter SIS mixers. Harmonic conversion effects must become less important as

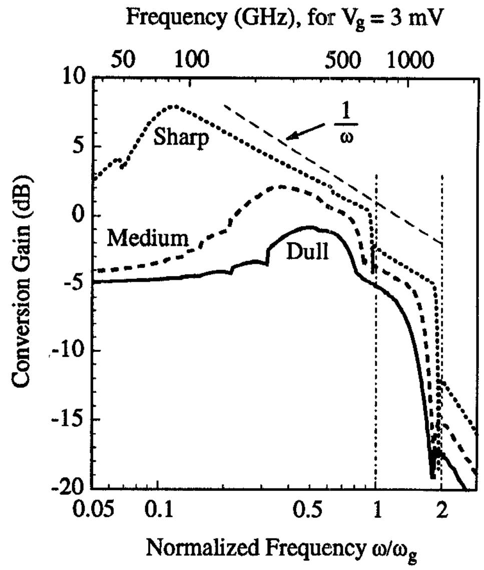

Fig. 7. The IF conversion gain corresponding to the three curves of Fig. 2.

the frequency is increased, because the SIS junction presents a weaker nonlinearity for harmonic frequencies above ω_g, especially so for frequencies above $2\omega_g$. Thus the beneficial effects of the capacitance are reduced as the frequency is increased, and smaller values of $\omega R_N C$ can be tolerated. There is evidence for this in [11]. Since a wider range of $\omega R_N C$ is acceptable, the choice must be made on purely technological grounds. It is more difficult to resonate the capacitance at high frequency, and so smaller values of $\omega R_N C$ are desirable. However, small area and high critical current SIS junctions are difficult to fabricate, and usually entail undesirable consequences such as inferior junction quality, poorer yield, etc. Therefore, the choice of $\omega R_N C$ for submillimeter SIS mixers will at best be an informed compromise.

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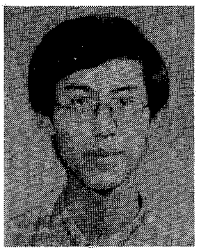

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